

DM TS n°4.

Ex 1.

$$\textcircled{1} \frac{V_{n+1}}{V_n} = \frac{4\left(\frac{1}{3}V_n + n - 1\right) - 6(n+1) + 15}{4V_n - 6n + 15} = \frac{\frac{4}{3}V_n - 2n + 5}{4V_n - 6n + 15} = \frac{1}{3}$$

donc (V_n) est géométrique de raison $q = \frac{1}{3}$ et de premier terme $V_0 = 4 - 6 \times 0 + 15 = 19$ $\textcircled{1}$

$$\textcircled{2} U_n = \frac{1}{4}U_n + \frac{3}{2}n - \frac{15}{4} = \frac{19}{4} \times \left(\frac{1}{3}\right)^n + \frac{6n - 15}{4} \quad \textcircled{1}$$

$\textcircled{3} t_n = -\frac{15}{4} + \frac{3}{2}n$ est une suite arithmétique de raison $\frac{3}{2}$ et de premier terme $-\frac{15}{4}$ $\textcircled{1}$

$w_n = \frac{19}{4} \times \left(\frac{1}{3}\right)^n$ est une suite géométrique de raison $\frac{1}{3}$ et de premier terme $\frac{19}{4}$

$$S_n = \frac{n+1}{2}(2t_0 + t_n) + w_0 \frac{1 - q^{n+1}}{1 - q} = \frac{n+1}{2} \left(-\frac{15}{2} + \frac{3}{2}n \right) + \frac{19}{4} \frac{1 - \left(\frac{1}{3}\right)^{n+1}}{\frac{2}{3}} \quad \textcircled{2}$$

II) Partie A

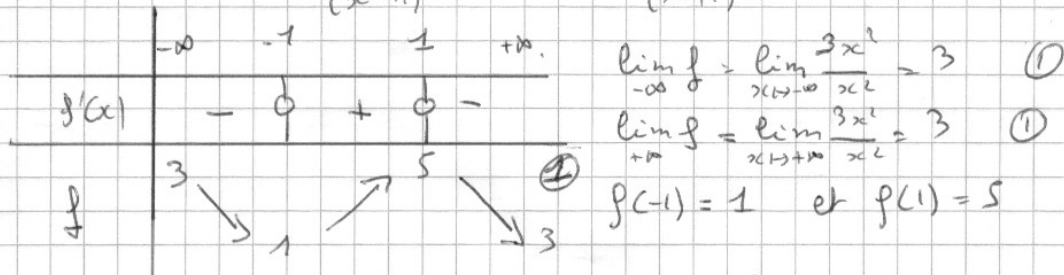
$$\Phi'(x) = \frac{(6x^2(x^2+1) - 2x(3x^2+ax+b))}{(x^2+1)^2} = \frac{6x^3 + 6x^2 + 6x^2ax + a - 6x^3 - 2ax^2 - 2bx}{(x^2+1)^2} = \frac{-ax^2 + (-2b+6)x + a}{(x^2+1)^2} \quad \textcircled{1}$$

$$\left. \begin{array}{l} \Phi(0) = 3 \Leftrightarrow b = 3 \\ \Phi'(0) = 4 \Leftrightarrow a = 4 \end{array} \right\} \Phi(x) = \frac{3x^2 + 4x + 3}{x^2 + 1} \quad \textcircled{1}$$

Partie B

$$1. d + \frac{\beta x}{x^2+1} = \frac{dx^2 + \beta x + d}{x^2+1} \quad \text{d'où } d = 3 \text{ et } \beta = 4. \quad \textcircled{1}$$

$$2. f'(x) = \Phi'(x) = \frac{-4x^2 + 4}{(x^2+1)^2} = \frac{-4(x^2-1)}{(x^2+1)^2} \quad \textcircled{1}$$



$$3. f(x) - (4x+3) = 3 + \frac{4x}{x^2+1} - 4x - 3 = \frac{-4x^3}{x^2+1} \quad \textcircled{1}$$

