Correction DM n°8

Ex1 :

① • $z_1 = \frac{\sqrt{6} - i\sqrt{2}}{2}$

$|z_1| = \sqrt{\frac{6}{4} + \frac{2}{4}} = \sqrt{\frac{8}{4}} = \sqrt{2}$

Soit $\theta = \mathrm{Arg}(z_1)\ (2\pi)$

$\left.\begin{array}{l} \cos\theta = \frac{\sqrt{3}}{2} \\ \sin\theta = -\frac{1}{2} \end{array}\right\} \theta = -\frac{\pi}{6}(2\pi)$

$z_1 = \sqrt{2}\, e^{i\frac{-\pi}{6}}$ (1,0)

• $z_2 = 1 - i$

$|z_2| = \sqrt{2}$

Soit $\theta' = \mathrm{Arg}(z_2)\ (2\pi)$

$\left.\begin{array}{l} \cos\theta' = \frac{1}{\sqrt{2}} \\ \sin\theta' = -\frac{1}{\sqrt{2}} \end{array}\right\} \theta' = -\frac{\pi}{4}(2\pi)$

$z_2 = \sqrt{2}\, e^{i\frac{-\pi}{4}}$ (1,0)

• $Z = \frac{z_1}{z_2} = \frac{\sqrt{2}\, e^{i\frac{-\pi}{6}}}{\sqrt{2}\, e^{i\frac{-\pi}{4}}} = e^{i\left(-\frac{\pi}{6} + \frac{\pi}{4}\right)} = e^{i\frac{\pi}{12}} \rightarrow \begin{cases} |Z| = 1 \\ \mathrm{Arg}(Z) = \frac{\pi}{12}\ (2\pi) \end{cases}$ (1,0)

② $Z = \frac{(\sqrt{6} - i\sqrt{2})(1+i)}{2(1-i)(1+i)} = \frac{\sqrt{6} + \sqrt{2}}{4} + \frac{\sqrt{6} - \sqrt{2}}{4} i$ (1)

Comme $\begin{cases} |Z| = 1 \\ \mathrm{Arg}(Z) = \frac{\pi}{12}\ (2\pi) \end{cases}$ On a : $\begin{cases} \cos\frac{\pi}{12} = \frac{\sqrt{6} + \sqrt{2}}{4} \\ \sin\frac{\pi}{12} = \frac{\sqrt{6} - \sqrt{2}}{4} \end{cases}$ (1)

③ $(\sqrt{6} + \sqrt{2})\cos x + (\sqrt{6} - \sqrt{2})\sin x = 2$

$4\cos\frac{\pi}{12}\cos x + 4\sin\frac{\pi}{12}\sin x = 2$

$\cos\left(\frac{\pi}{12} - x\right) = \frac{1}{2}$ (1)

$\cos\left(\frac{\pi}{12} - x\right) = \cos\left(\frac{\pi}{3}\right)$

ou $\frac{\pi}{12} - x = \frac{\pi}{3}\ (2\pi)$ ou $\frac{\pi}{12} - x = -\frac{\pi}{3}\ (2\pi)$ (1)

ou $x = -\frac{\pi}{4}(2\pi)$ ou $x = \frac{5\pi}{12}(2\pi)$ $\longrightarrow$ $\mathcal{S} = \left\{ -\frac{\pi}{4}(2\pi) ; \frac{5\pi}{12}(2\pi) \right\}$ (1)

Ex2

$(E) : y' = 2y + e^{2x}$

① • $u(x) = xe^{2x} \rightarrow u$ est dérivable sur $\mathbb{R}$ et $u'(x) = e^{2x} + 2xe^{2x} = e^{2x}(1 + 2x)$

• $2u + e^{2x} = 2xe^{2x} + e^{2x} = u' \rightarrow u$ vérifie E sur $\mathbb{R}$. (1)

② $(E_0) : y' = 2y + 0 \rightarrow$ Toute solution de cette équation s'écrit sous la forme $f(x) = Ke^{2x}$ où $K \in \mathbb{R}$ (1)

③ $v - u$ solution de (E_0)

$\Leftrightarrow (v - u)' = 2(v - u)$

$\Leftrightarrow v' - u' = 2v - 2u$

$\Leftrightarrow v' = 2v - 2u + u'$

$\Leftrightarrow v' = 2v - 2xe^{2x} + e^{2x} + 2xe^{2x}$

$\Leftrightarrow v' = 2v + e^{2x}$

$\Leftrightarrow v$ solution de (E) ②

4) les solutions de (E) sont donc de la forme

$v(x) = Ke^{2x} + xe^{2x}$ ①

5) $y(0) = 1$

$v(0) = K = 1$ donc $v(x) = e^{2x} + xe^{2x} = (x+1)e^{2x}$ ①

6) a) . $D_f = \mathbb{R}$.

. le produit de 2 fo derivables est derivable donc f est derivable sur $\mathbb{R}$. 0,1

$f'(x) = e^{2x} + (2x+2)e^{2x} = e^{2x}(3+2x)$ ①

x	$-\infty$		$-\frac{3}{2}$		$+\infty$
$3+2x$		$-$	0	$+$	
$f(x)$		$-$	0	$+$	
f	0	$\searrow$	$\frac{e^{-3}}{-2}$	$\nearrow$	$+\infty$

①

$\left.\begin{array}{l} \lim\limits_{x \to +\infty} x+1 = +\infty \\ \lim\limits_{x \to +\infty} e^{2x} \end{array}\right\} \lim\limits_{+\infty} f = +\infty$ 1,5

$f(x) = xe^{2x} + e^{2x} = \frac{1}{2}(2x)e^{2x} + e^{2x}$

$\left.\begin{array}{l} \lim\limits_{x \to -\infty} 2xe^{2x} = 0 \\ \lim\limits_{x \to -\infty} e^{2x} = 0 \end{array}\right\} \lim\limits_{-\infty} f = 0$ 2