

DM n°5

(I) a) $z \neq 0$ * $z^2 + 1 = z$ soit $z^2 - z + 1 = 0$. $\Delta = -3 \rightarrow \begin{cases} z_1 = \frac{1+i\sqrt{3}}{2} \\ z_2 = \frac{1-i\sqrt{3}}{2} \end{cases}$ $S = \left\{ \frac{1+i\sqrt{3}}{2}, \frac{1-i\sqrt{3}}{2} \right\}$

b) $\Delta = 4(1+\cos\theta)^2 - 4(1+\cos\theta) \times 2 = 4+4\cos\theta+8\cos\theta-8-8\cos\theta = 4(\cos\theta-1) = -4\sin^2\theta$

• si $\theta = 0(\pi)$ alors $\Delta = 0$ et $S = \left\{ \frac{1+\cos\theta}{2} \right\}$

• si $\theta \neq 0(\pi)$ alors $\Delta < 0$: $i\sqrt{\Delta} = 2i|\sin\theta|$: $S = \left\{ \frac{1+\cos\theta}{2} + \frac{i|\sin\theta|}{2}, \frac{1+\cos\theta}{2} - \frac{i|\sin\theta|}{2} \right\}$

Comme $\sin\theta = \pm|\sin\theta|$, $S = \left\{ \frac{1+\cos\theta+i\sin\theta}{2}, \frac{1+\cos\theta-i\sin\theta}{2} \right\}$

c) On pose $Z = z^2$. on doit résoudre $Z^2 + 4Z + 3 = 0$

$\Delta = 16 - 12 = 4 \rightarrow \begin{cases} Z_1 = \frac{-4+2}{2} = -1 \\ Z_2 = \frac{-4-2}{2} = -3 \end{cases}$

Soit $z^2 = -1$ soit $z^2 = -3 \rightarrow S = \{i; -i; \sqrt{3}i; -\sqrt{3}i\}$

(II) ① $f =]-\infty; -1[\cup]0; +\infty[$

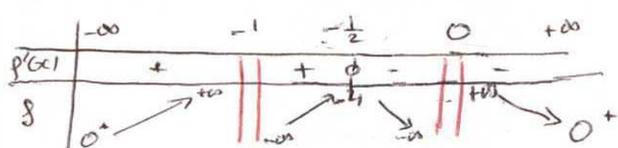
une f rationnelle est dérivable et continue sur chaque intervalle où elle est définie

② $f'(x) = \frac{-(x(x+1))'}{x^2(x+1)^2} = \frac{-(x+1) - x}{x^2(x+1)^2} = \frac{-2x-1}{x^2(x+1)^2}$

$\lim_{x \rightarrow -\infty} f = \lim_{x \rightarrow +\infty} \frac{1}{x(x+1)} = 0^+$ et $\lim_{x \rightarrow +\infty} f = \lim_{x \rightarrow +\infty} \frac{1}{x(x+1)} = 0^+$: asymptote horizontale $y = 0$ en $+\infty$

$\lim_{x \rightarrow -1^-} f = +\infty$ $\lim_{x \rightarrow -1^+} f = -\infty$: asymptote verticale $x = -1$

$\lim_{x \rightarrow 0^-} f = -\infty$ $\lim_{x \rightarrow 0^+} f = +\infty$: asymptote verticale $x = 0$



$f(-\frac{1}{2}) = \frac{1}{(\frac{1}{2}+1) \cdot \frac{1}{2}} = \frac{1}{-\frac{1}{4}} = -\frac{4}{3}$

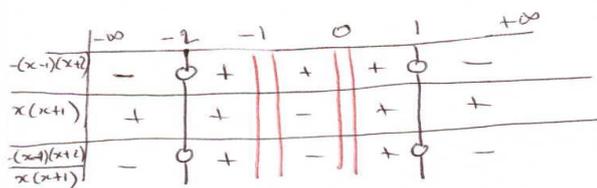
③ $f(x)$ est positif sur $]-\infty; -1[\cup]0; +\infty[$ et négatif sur $]-1; 0[$

④ $\frac{1}{(x+1)x} \leq \frac{1}{2}$ pour $x \in \mathbb{D}$

$\frac{1}{x(x+1)} - \frac{1}{2} \leq 0$

$\frac{2 - x^2 - 2x}{x(x+1)} \leq 0$

$\frac{-(x-1)(x+2)}{x(x+1)} \leq 0$



D'où $S =]-\infty; -2] \cup]-1; 0[\cup]1; +\infty[$

$$\textcircled{5} \frac{a}{x} + \frac{b}{x+1} = \frac{(a+b)x + a}{x(x+1)} = \frac{0x+1}{x(x+1)} \rightarrow \begin{cases} a=1 \\ b=-1 \end{cases} : f(x) = \frac{1}{x} - \frac{1}{x+1}$$

$$\textcircled{6} \left. \begin{array}{l} U_n = f(n) \\ \lim_{n \rightarrow +\infty} f = 0 \end{array} \right\} (U_n) \text{ converge vers } 0$$

$$\begin{aligned} \textcircled{7} S_n &= f(1) + f(2) + \dots + f(n) \\ &= \frac{1}{1} - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \dots + \frac{1}{n} - \frac{1}{n+1} \\ &= 1 - \frac{1}{n+1} \\ &= \frac{n}{n+1} \rightarrow \lim S_n = \lim \frac{n}{n+1} = 1 \end{aligned}$$

III

$$\textcircled{1} \lim \frac{n\pi}{2n+1} = \lim \frac{n\pi}{2n} = \frac{\pi}{2}$$

$$\textcircled{2} a. \lim_{x \rightarrow \frac{\pi}{2}} \cos(x) = 0$$

$$b. \lim \cos\left(\frac{n\pi}{2n+1}\right) = 0$$

IV 1. Soit z l'affixe de G et z' celle de G'

$$z = \frac{1}{3} (2+i - 1-i + 4+3i)$$

$$z = \frac{1}{3} (3+3i)$$

$$z = 1+i$$

$$z' = \frac{1}{7} (2(2+i) + (-1-i) - 2(2+3i))$$

$$= \frac{1}{7} (4+2i - 1-i - 4-6i)$$

$$= -1-5i$$

$$2. \frac{\|\vec{MA} + \vec{MB} + \vec{MC}\|}{\|2\vec{MA} + \vec{MB} - 2\vec{MC}\|} = 3 \Leftrightarrow \frac{\|3\vec{MG}'\|}{\|\vec{MG}'\|} = 3 \Leftrightarrow \frac{MG}{MG'} = 1 \Leftrightarrow MG = MG'$$

Le lieu de M est la médiatrice de $[GG']$